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BAUER'S UNSOLVABLE ACCUMULATION SCHEMES
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In this paper we give a new statement on Bauer's (1913) accumulation schemes. It is an important model because it is one of the first models where the question of capital transfers is explicitly introduced. By using and extending Orzech & Groll (1983) formalization of the problem we are able to make three definitive statements. First, we clarify the debate engaged between O. Bauer and R. Luxemburg (1913) on the realisation of surplus value in a capitalist economy in which the development of the productive forces is taken into account. This enables us to give a definite answer on the possibility of a boundless economic accumulation in a capitalist society. Second, we analyse and criticize Bauer's crisis theory. Finally, we show that the over determination of Bauer's model, first demonstrated by Bronfenbrenner & Wolfson (1984) in an Harrodian framework, cannot be lifted up by allowing the rate of surplus to increase from period to period, as Samuelson & Wolfson (1986) think, using Bronfenbrenner & Wolfson's (1984) formulation of the problem. The reason for this impossibility lays in the fact that this model is a bisectorial one where the markets are, by assumptions, cleared at every period at the same time as the rate of exploitation and the pace of technical progress are given. This conclusion could be extended to many Marxist models, notably the one by Luxemburg (1913).

In a first part, we will present Bauer numerical example and assumptions derived from it, explicitly or implicitly. Orzech & Groll (1983) formalization of Bauer's model is extended, allowing to determine capital transfers at each period.

In a second part, we present what could be called the proportion crisis in Bauer's model. The possibility of this crisis was first discovered by R. Luxemburg in her critical evaluation of Marxist analysis of capitalist development. Bauer's boundless capitalist accumulation would not be possible because of an impossibility to realize part of the surplus value. Bauer, in fact, reasons in a socialist society where production is organized by a central

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organ. Surplus value embodied in productions, in this context, has not to be realized before being invested. This leads us to a second point on the interpretation of the development of productive forces. In Luxemburg's model, and, in general, in the Marxist tradition, sectoral organic compositions of capital increase at the same speed that the global organic composition of capital. In this sense it can be said that they represent the mechanization of production. In Bauer's model, they do not. They adapt to physical capital transfers between sectors because markets cannot clear without this amendment. They do not represent the mechanization of production. With this interpretation of the development of productive forces it seemed logical, because the exchange of money against goods cannot be treated in the schemes, that Bauer thought he answered to Luxemburg's criticism. Therefore, if Luxemburg is wrong when she excludes capitalist competition from her scheme of extended reproduction, Bauer is also wrong in considering he overcame the difficulty. He just showed the possibility of accumulation in a socialist society. That is why Bauer's formulation of macroeconomic crises and fluctuations in a capitalist society cannot be accepted. But, continuing to develop Orzech & Groll's (1983) formalization we discover that the centrally planned economy runs out of surplus value after 34 periods if the rhythm of technical development is imposed. We cannot speak anymore of capital transfers between sectors. Technical development coupled with the assumption that markets clear at every period imposes that the sector producing consumption goods do not transfer goods anymore but make that a part of its disinvestment, necessary to respect the desired increase of the global organic composition of capital, is supported by the sector producing capital goods. This will last until the sector producing consumer goods disappears. We can therefore conclude that even a socialist economy accumulation of capital is not necessarily boundless.

To overcome this difficulty, it seemed that we had to allow the rate of surplus value or the rate of exploitation to increase in time. But a simple demonstration shows that this cannot be done without contradicting the assumption that markets clear at each period. There would be an excess supply of consumption goods. This could not be seen by Brofenbrenner & Wolfson (1984) and Samuelson & Wolfson (1986) because they transformed Bauer's bisectorial model into a unisectorial one. Finally, we show that the rate of surplus cannot be even fixed at a level which would allow the economy to run out of profit at the latest times. In fact, the assumption that markets clear at each period coupled with the fact that the rhythm of the growth of global organic composition is imposed, make the model over determined. In this case, either the rate of surplus value is given and the increase in the global organic composition of capital is endogenous or, the increase of global organic composition is given,

i.e. technical development, and the rate of surplus value, because markets have to clear systematically, become endogenous. This negative conclusion seems to be true of Luxemburg's model too. This critical evaluation can find an echo in the rhetoric used by Bauer when he presents the numbers in his model as "arbitrary assumptions". We know that assumptions have to be justified and numbers chosen arbitrarily.

I. Bauer's model:

A. Bauer's numerical example:

According to Luxemburg, "*Bauer's own assumptions are quite laudable*" and "*there is not event room there for one more nail*" (Luxemburg (1913), p. 92-93). Bauer's assumptions are sometimes explicit and, at other times, can be deduced from his tables:

1) Variable capital grows at 5% (β) per period;
2) Constant capital grows at 10% (α) per period:

   "*variable capital must... grow at 5 per cent each year. Constant capital increases faster than variable capital. (...) We will assume that constant capital grows at 10 per cent per annum*" (Bauer (1913), p. 92-93).

Consequently, the global organic composition of capital increases from period to period. This growth represents the "development of productive forces", the development of the mechanisation of the total production. From Bauer's numerical example we obtain:

	C_t	V_t
Period 0	200000	100000
Period 1	$220000 = (1+\alpha)C_0$	$105000 = (1+\beta)V_0$
Period 2	$242000 = (1+\alpha)^2 C_0$	$110250 = (1+\beta)^2 V_0$
Period 3	$266200 = (1+\alpha)^3 C_0$	$115762 = (1+\beta)^3 V_0$

3) The rate of surplus value, μ, is constant and equal in the two sectors (100%).
4) Global capitalists' consumption out of profits, DK_t, is residual. From Bauer's numerical example we have:

	Π_t	DK_t	αC_t	βV_t
Period 0	$100000 = \mu V_0$	75000	20000	5000
Period 1	$105000 = \mu V_1$	77750	22000	5250
Period 2	$110250 = \mu V_2$	80539	24200	5511
Period 3	$115762 = \mu V_3$	83374	26600	5788

5) The rate of savings out of profits, s_t, called accumulation rate by Bauer, is identical in both departments:

"*the rate of accumulation is the same in both departments*"[1] (*ibid.*, p. 96).

6) Total surplus value accumulated, $AC_{j,t} + AV_{j,t}$ (where j = I, II), is divided in each sector between constant and variable capitals in equal proportions to the ratio between constant capital and variable capital newly accumulated in the economy.

Assumptions 5) and 6) imply that total surplus value is accumulated in equal proportions to the ratio between constant capital and variable capital newly accumulated in the economy[2]. The next table, for period $t = 0$, is given by Bauer:

	Sector I (Capital goods.)	Sector II (Cons. goods)	Total
$C_{j,0}=C_{j,-1}+IC_{j,-1}$	120000	80000	$200000 = C_0$
$V_{j,0}=V_{j,-1}+IV_{j,-1}$	50000	50000	$100000 = V_0$
$DK_{j,0}$	37500	37500	$75000 = DK_0$
$AC_{j,0}$	10000	10000	$20000 = \alpha C_0$
$AV_{j,0}$	2500	2500	$5000 = \beta V_0$
Total	220000	180000	400000

The surplus value in each sector is not necessarily invested in the sector in which it has been created. It is the transfer process. Net investments in constant and variable capitals in sector I and II, $IC_{II,t}$, $IC_{I,t}$, $IV_{II,t}$ et $IV_{I,t}$, are different from accumulations in constant and variable capitals in sectors I and II.

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[1] We can here assert that there exists a difference with the functioning of the extended accumulation schemes of Marx and Luxemburg. In these models, capitalists of department I (producing capital goods) decide their rates of savings whereas those of department II (producing consumption goods) adapt their own rates of savings, given sectoral organic compositions of capital.

[2] $(AC_{I,t} + AC_{II,t})/(AV_{I,t} + AV_{II,t}) = \alpha C_t / \beta V_t$.

7) Bauer (*ibid.*, foot note p. 95) makes the assumption that the ratio between net investments in constant and variable capitals in sector I is equal to the ratio between constant capital and variable capital newly accumulated in the economy:

$$IC_{II,t} \,/\, IV_{II,t} = \alpha C_t \,/\, \beta V_t.$$

Furthermore, we can notice that assumptions 5), 6) and 7) imply that,

$$IC_{I,t} \,/\, IV_{I,t} = \alpha C_t \,/\, \beta V_t.$$

The last assumption, coupled with the fact that all the surplus value accumulated is invested, insures the clearing of the markets. However, the fact that capitalists are informed of the growth of the global organic composition of capital does not mean that sectoral organic compositions of capital will grow at the same pace.

To see this last point, we must calculate capital transfers between sectors in order to determine the evolutions of sectoral organic compositions of capital.

From assumptions 1), 2), 3), and 4), demands to sector I and II are given. Values of total productions at time $t = 1$, in sectors I and II, $P_{j,t}$, are consequently fixed:

$$DK_1 + V_1 + \beta V_1 = P_{II,1},$$

which gives,

$$77750 + 105000 + 5250 = 188000 \,;$$

$$C_1 + \alpha C_1 = P_{I,1},$$

which gives,

$$220000 + 22000 = 242000.$$

We have to notice that if all the surplus value accumulated in each sector was invested in the sector in which it was created, the amount of the production of sector I at time $t = 1$ would be 235000[3]. The one of sector II at time $t = 1$ would be 195000[4].

There must therefore be a surplus value transfer, from sector II to sector I, for the supplies to adjust to demands, determined by assumptions:

> "*It is impossible for the surplus value accumulated in the two departments of production to be invested productively in the same departments in the following year, for the progress to a higher organic composition of capital requires a movement of capital from the consumer-goods industries into the production of means of production. Thus we must calculate what part of the surplus value accumulated in the consumer-goods industries must be transferred to the*

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[3] $120000 + 10000 + 2.[(50000 + 2500)] = 235000.$
[4] $80000 + 10000 + 2.[(50000 + 2500)] = 195000.$

production of means of production to be used for their expansion. (…) In the second year the value of the products of consumer-goods industries must thus amount to 188000, for the consumer goods can only be exchanged against these sums of value" (*ibid.*, p. 94-95).

"The capitalists in the consumer-goods industries transfer part of the surplus value accumulated in the first year to the means-of-production industries – either by themselves establishing factories for the production of means-of-production, or by transferring part of their accumulated surplus-value, through the mediation of the banks, for use of the capitalists in the means-of-production industries; or through buying shares in companies which produce means of production" (*ibid.*, p. 98).

With $IC_{II,0}$ and $IV_{II,0}$, respectively, net investments in constant and variable capitals in sector II at time $t = 1$, we have,

$$80000 + IC_{II,0} + 2.[(50000 + IV_{II,0})] = 188000.$$

Moreover, with assumption 7) indicating that,

$$IC_{II,0} / IV_{II,0} = \alpha C_0 / \beta V_0,$$

we find:

$$IC_{II,0} = 5334, \text{ and,}$$

$$IV_{II,0} = 1333.$$

Writing, respectively, $TC_{I,0}$ et $TV_{I,0}$, constant and variable capital transfers from sector II to sector I at time $t = 0$, sector II must transfer:

$$TC_{I,0} = AC_{II,0} - IC_{II,0} = 10000 - 5334 = 4666 \text{ and,}$$

$$TV_{I,0} = AV_{II,0} - IV_{II,0} = 2500 - 1333 = 1167,$$

from its accumulated surplus-value in constant and variable capitals.

Net investments in constant and variable capitals in sector I will be:

$$IC_{I,0} = \alpha C_0 - IC_{II,0} ;$$

$$IV_{I,0} = \beta V_0 - IV_{II,0} ;$$

or,

$$IC_{I,0} = AC_{I,0} + TC_{I,0} ;$$

$$IV_{I,0} = AV_{I,0} + TV_{I,0}.$$

It is however possible to determine directly net investments in constant and variable capital in sector I.

With,

$$120000 + IC_{I,0} + 2.[(50000 + IV_{I,0})] = 242000, \text{ and,}$$

$$IC_{I,0} / IV_{I,0} = \alpha C_0 / \beta V_0,$$

we find,

$$IV_{I,0} = 3667, \text{ and,}$$

$$IC_{I,0} = 14666.$$

Consequently,

$$IV_{II,0} = \beta V_0 - IV_{I,0} = 5000 - 3667 = 1333$$

$$IC_{II,0} = \alpha C_0 - IC_{I,0} = 20000 - 14666 = 5334.$$

Bauer can therefore write for period $t = 1$:

	Sector I (Capital goods)	Sector II (Cons. goods)	Total
$C_{j,1}=C_{j,0}+IC_{j,0}$	134666	85334	$220000 = C_1$
$V_{j,1}=V_{j,0}+IV_{j,0}$	53667	51333	$105000 = V_1$
$DK_{j,1}$	39740	38010	$77750 = DK_1$
$AC_{j,1}$	11244	10756	$22000=\alpha C_1$
$AV_{j,1}$	2683	2567	$5250=\beta V_1$
Total	242000	188000	430000

Capital transfers between sectors, or the fact that the surplus value accumulated in each sector is not totally invested in the sector in which it was created, allows the adjustment of the supply structure to the demand structure. The economy is therefore in desequilibrium whereas markets clear at each period.

B. A generalisation:

Capital transfers are determined in order to make possible the adjustment of supply to demand on each market. Bauer develops his numerical example over 4 periods. But it is possible to generalise Bauer's method to know capital transfers and net investments in constant and variable capitals for each sector at every period.

We know that sector II produces all consumption goods[5]. Demand of consumption goods at each period is,

$$P_{II}(t) = V(t) + \beta V(t) + DK(t).$$

We also know that,

$$DK(t) = \Pi(t) - \beta V(t) - \alpha C(t) \text{ or } DK(t) = \mu V(t) - \alpha C(t) - \beta V(t),$$

where μ is the rate of surplus value (100%).

We obtain,

$$P_{II}(t) = (1 + \mu) V(t) - \alpha C(t).$$

The next period,

$$P_{II}(t+1) = (1 + \mu) V(t+1) - \alpha C(t+1) = (1 + \mu)(1 + \beta) V(t) - \alpha(1 + \alpha) C(t).$$

We can therefore write,

$$P_{II}(t+1) - P_{II}(t) = \Delta P_{II}(t+1) = (1 + \mu) \beta V(t) - \alpha^2 C(t).$$

On the supply side, we know, by assumptions, that,

$$P_{II}(t) = C_{II}(t) + V_{II}(t) + \Pi_{II}(t) \text{ or } P_{II}(t) = C_{II}(t) + (1 + \mu) V_{II}(t),$$

But we also know that the change in output in department II is :

$$\Delta P_{II}(t+1) = IC_{II}(t) + (1 + \mu) IV_{II}(t),$$

where, $IC_{II}(t)$ is the net investment in constant capital in sector II once capital transfers have taken place, and $IV_{II}(t)$ is the net investment in variable capital in sector II once capital transfers have taken place.

If the variations in the production of consumption goods must be equal to the variations of consumption goods demanded, we have,

$$(1 + \mu) \beta V(t) - \alpha^2 C(t) = IC_{II}(t) + (1 + \mu) IV_{II}(t).$$

Assumption 7),

$$IC_{II}(t) / IV_{II}(t) = \alpha C(t) / \beta V(t),$$

allows us to determine,

$$IV_{II}(t) = \beta V(t) \frac{(1 + \mu) \beta V(t) - \alpha^2 C(t)}{(1 + \mu) \beta V(t) + \alpha C(t)} \quad (1),$$

$$IC_{II}(t) = \alpha C(t) \frac{(1 + \mu) \beta V(t) - \alpha^2 C(t)}{(1 + \mu) \beta V(t) + \alpha C(t)} \quad (2).$$

Recalling that,

$$V(t) = V(0).(1 + \beta)^t \text{ and,}$$

$$C(t) = C(0).(1 + \alpha)^t,$$

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[5] This presentation is largely inspired by Orzech & Groll (1983).

it is now possible to determine all net investments in constant and variable capitals at each period[6] :

$$IV_{II}(t) = \beta V(0)\,(1+\beta)^t\ \frac{(1+\mu)\,\beta V(0)\,(1+\beta)^t\ -\ \alpha^2 C(0)\,(1+\alpha)^t}{(1+\mu)\,\beta V(0)\,(1+\beta)^t\ +\ \alpha C(0)\,(1+\alpha)^t}\ ;$$

$$IC_{II}(t) = \alpha C(0)\,(1+\alpha)^t\ \frac{(1+\mu)\,\beta V(0)\,(1+\beta)^t\ -\ \alpha^2 C(0)\,(1+\alpha)^t}{(1+\mu)\,\beta V(0)\,(1+\beta)^t\ +\ \alpha C(0)\,(1+\alpha)^t}\ ;$$

$$IV_I(t) = \beta V(0)\,(1+\beta)^t - IV_{II}(t)\ ;$$

$$IC_I(t) = \alpha C(0)\,(1+\alpha)^t - IC_{II}(t).$$

Let us notice that $\alpha C(0)\,(1+\alpha)^t$ et $\beta V(0)\,(1+\beta)^t$ and therefore, $(1+\mu)\,\beta V(0)\,(1+\beta)^t + \alpha C(0)\,(1+\alpha)^t$, increase in time, whereas $(1+\mu)\,\beta V(0)\,(1+\beta)^t - \alpha^2 C(0)\,(1+\alpha)^t$ increases on the first 20 periods and decreases before becoming negative[7] at period $t = 35$. This point will be analysed later. Troubles beginning at period $t = 34$, when saving is insufficient to allow the global organic composition of capital to increase, such that constant capital grows by 10% per period and variable capital grows by 5%, we will first concentrate on the 34 first periods ($t = 33$).

It is consequently possible to know the evolutions of gross investments in constant and variable capitals, $C_j(t)$ et $V_j(t)$ ($j = I, II$), in each sector and for every period. They are of the type $u(n) = f(n ; u(n-1))$.

$$C_I(t) = C_I(t-1) + IC_I(t\text{-}1)\,;\ \text{with}\ C_I(0) = 120000.$$

$$V_I(t) = V_I(t-1) + IV_I(t\text{-}1)\,;\ \text{with}\ V_I(0) = 50000.$$

$$C_{II}(t) = C_{II}(t-1) + IC_{II}(t\text{-}1)\,;\ \text{with}\ C_{II}(0) = 80000.$$

$$V_{II}(t) = V_{II}(t-1) + IV_{II}(t\text{-}1)\,;\ \text{with}\ V_{II}(0) = 50000.$$

We can also find capital transfers for each period. We know that,

$$TC_I(t) = AC_{II}(t) - IC_{II}(t),$$

$$TV_I(t) = AV_{II}(t) - IV_{II}(t).$$

Assumption 7) stipulates that,

$$AC_{II}(t)\,/\,AV_{II}(t) = \alpha C(t)\,/\,\beta V(t),$$

whereas,

$$AC_{II}(t) + AV_{II}(t) = s(t)\,V_{II}(t).$$

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[6] The same results would have followed if we had taken sector I as example. With $IC_I(t) + (1+\mu)\,IV_I(t) = C(t)\,(\alpha + \alpha^2) = \Delta P_I(t+1)$ and $IC_I(t)/IV_I(t) = \alpha C(t)\,/\,\beta V(t)$, recalling $V(t) = V(0)(1+\beta)^t$ and $C(t) = C(0)\,(1+\alpha)^t$, we obtain $IV_I(t) = \{[C(0)(1+\alpha)^t(\alpha + \alpha^2)]/[((\alpha\,C(0)(1+\alpha)^t)/(\beta\,V(0)(1+\beta)^t)) + (1+\mu)]\}$, and so on…

[7] $10000(1,05)^t - 2000(1,1)^t = 0$ implies that $t = [(\ln 5)/\ln(1,1/1,05)] = 34,\dots..$

From the last two equations, we can determine $AC_{II,t}$ and $AV_{II,t}$:

$$AC_{II}(t) = [s(t)].[V_{II}(t)]. \frac{\alpha C(t)}{\alpha C(t) + \beta V(t)} \qquad (3),$$

$$AV_{II}(t) = [s(t)].[V_{II}(t)]. \frac{\beta V(t)}{\alpha C(t) + \beta V(t)} \qquad (4),$$

with,

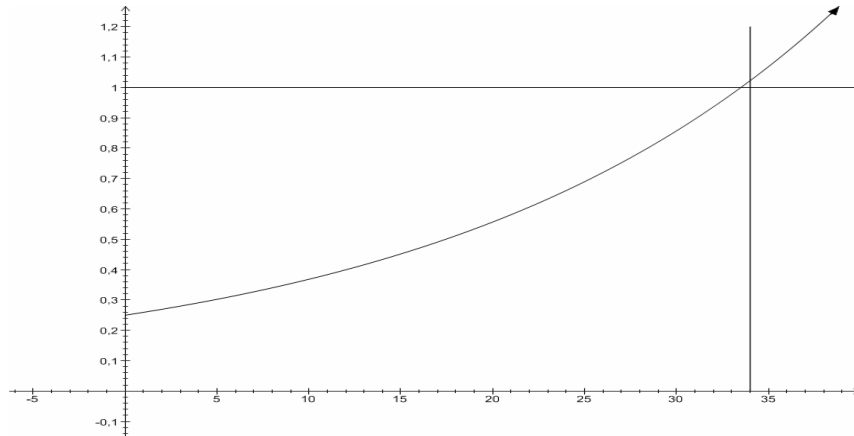$$s(t) = \frac{\alpha C(t) + \beta V(t)}{V(t)} \text{, and,}$$

$$V_{II}(t) = V_{II}(t-1) + IV_{II}(t-1) \text{ ; with } V_{II}(0) = 50000.$$

Thereby, evolutions of constant and variable capital transfers are:

$$TC_I(t) = \frac{\alpha C(0)(1+\alpha)^t}{\alpha C(0)(1+\alpha)^t + \beta V(0)(1+\beta)^t} .[s(t)].[V_{II}(t)] - IC_{II}(t),$$

$$TV_I(t) = \frac{\beta V(0)(1+\beta)^t}{\alpha C(0)(1+\alpha)^t + \beta V(0)(1+\beta)^t} .[s(t)].[V_{II}(t)] - IV_{II}(t).$$

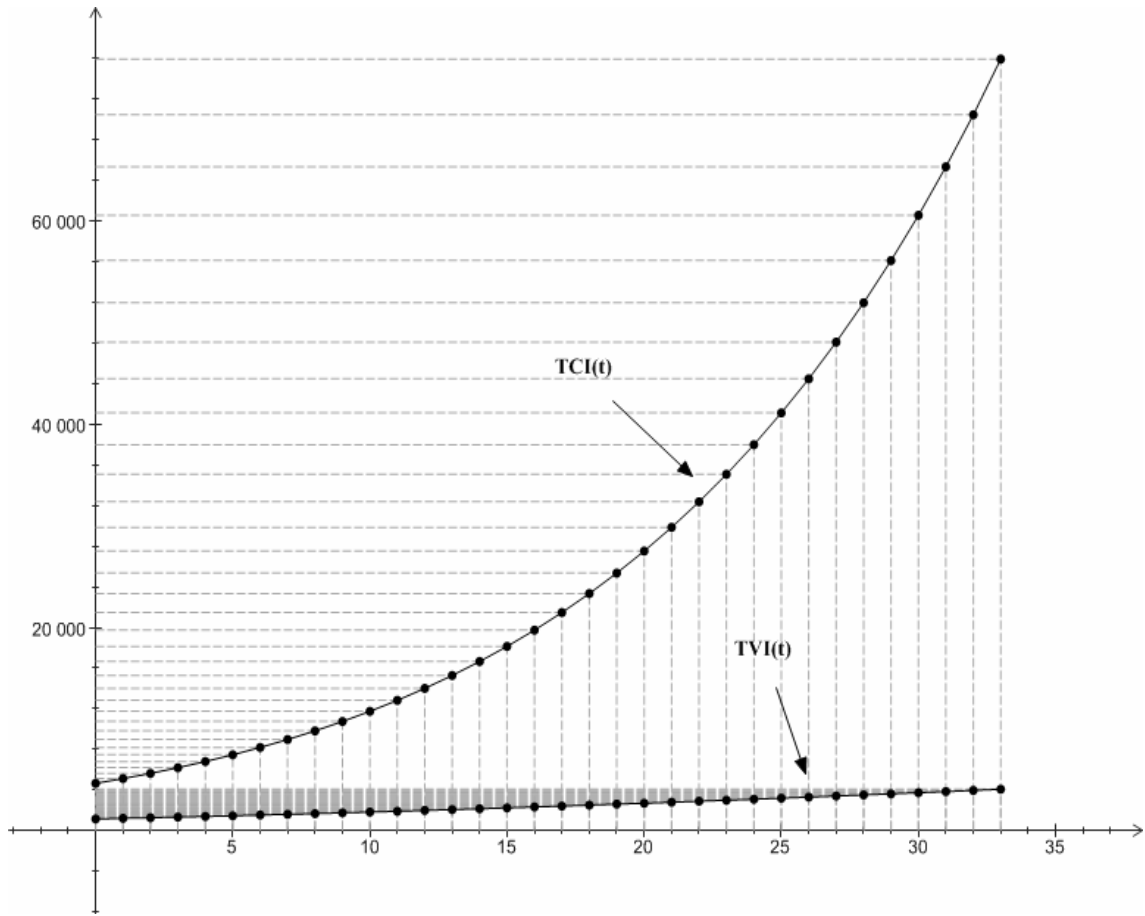Until period $t = 33$, the rate of saving out of profits, $s(t)$, is below 1,

and the formula can be rewritten,

$$TC_I(t) = \frac{\alpha C(0)(1+\alpha)^t}{V(0)(1+\beta)^t} .[V_{II}(t)] - IC_{II}(t),$$

$$TV_I(t) = \beta V_{II}(t) - IV_{II}(t).$$

Graphically,

Transfers in constant capital increase more than transfers in variable capital. It is here possible to reveal a property of the model. The ratio $TC_I(t)/TV_I(t)$ is always equal to the ratio between constant capital and variable capital newly accumulated in the economy, $\alpha C(t) / \beta V(t) = \dfrac{\alpha C(0)\,(1+\alpha)^t}{\beta V(0)\,(1+\beta)^t}$. We know that,

$$IC_{II}(t) /\ IV_{II}(t) = \alpha C(t) / \beta V(t),$$
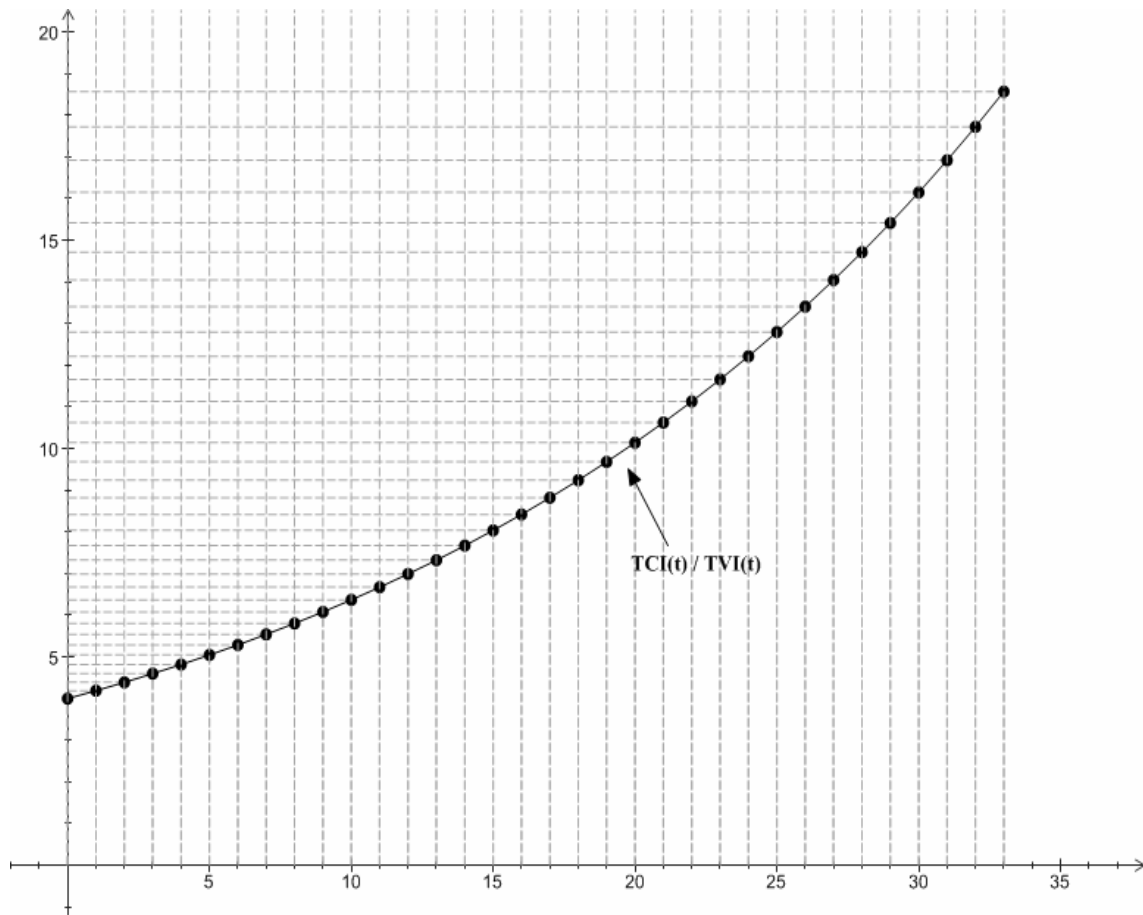$$AC_{II}(t) / AV_{II}(t) = \alpha C(t) / \beta V(t),$$

whereas,

$$AC_{II}(t) - IC_{II}(t) = TC_I(t),$$
$$AV_{II}(t) - IV_{II}(t) = TV_I(t).$$

We obtain therefore,

$$\frac{TC_I(t)}{TV_I(t)} = \frac{\alpha C(t)}{\beta V(t)}\left[\frac{AV_{II}(t) - IV_{II}(t)}{AV_{II}(t) - IV_{II}(t)}\right].$$

Graphically,

Let's now analyse Luxemburg's criticism on the possibility of a disproportion crisis and, in a next section, we will investigate the macroeconomic crisis in Bauer's model.

II. <u>Realisation of surplus-value and crisis theories:</u>

The debate between Bauer and Luxemburg is focused on the realisation of surplus-value:

> "*We now wish to discover whether the mass of commodities in which the accumulated part of surplus value is embodied can be sold within the capitalist world itself, or whether, as Comrade Luxemburg believes, it can find a market only outside the capitalist world*" (Bauer (1913), p. 97).

The core of the discussion is the possibility of extended reproduction in a capitalist economy. We will first present the terms of the debate, comments coming later.

At each period the production of sector I is $P_I(t)$. A part of it, $C_I(t)$, is devoted to replace constant capital used in this sector during the preceding period. An other, $AC_I(t)$, will be used

to extend the production of this sector at the next period. C_{II} (t) is devoted to replace constant capital used in sector II and IC_{II} (t) will be used to extend the production of department II at the next period. There is a remainder of constant capital in sector I which amounts to TC_I (t).

On the other side, at each period the production of sector II is P_{II} (t). A part of it, DK_{II} (t), is consumed by capitalists of sector II. An other, V_{II} (t), is devoted to replace variable capital used in this sector during the preceding period. A third, IV_{II} (t), will be used to extend the production of department II at the next period. V_I (t) is devoted to replace variable capital used in sector I during the preceding period. AV_I (t) will be used to extend the production of sector I at the next period and DK_I (t) is consumed capitalists of sector I. There is a remainder of constant capital in sector I which amounts to TV_I (t).

The solution suggested by Bauer is the following one:

> "*Thus the means-of-production industries buy commodities with a value of 4666 [= TC_I (0)] with that capital which was accumulated in the consumer-goods industries. And so the capitalists in the consumer-goods industries buy, in addition to means-of-production with a value of 85334 [= C_{II} (0) + IC_{II} (0)] for use in the production of consumer goods, additional means-of-production with a value of 4666 destined for the production of means-of-production.(…) Finally the capitalists in II use part of the surplus value they have accumulated for the construction of new factories, in which means-of-production are produced. These factories too must establish company stores, which have to buy part of the consumer goods produced in the first year in order to supply the requirements of the workers employed in the new factories in the second year. They purchase consumer goods with a value of 1167 [= TV_I (0)]*" (*ibid.*, p. 98-99).

According to him, this should prove

> "*that the accumulated part of surplus value can also be realised within the capitalist sphere itself*" (*ibid.*, p. 104).

To this, Luxemburg answers:

> "*Capitalists I "sell" their commodity-remainder of 4666 to capitalists II who "buy" it by transferring "part of their accumulated surplus value" to Dept. I. But wait a minute! What do they "buy" it with? Where is the "part of the surplus value" which pays for the purchase? There is no trace of it in Bauer's tables! The entire amount of commodities in Dept. II has already been used for the consumption of the capitalist class of both departments as well as for the renewal and enlargement of variable capital, at least except a remainder of 1167. This*

1167 in consumer goods is all that is left over from the surplus value of the second department.(…) Whichever way you look at the thing, the capitalists II have used up all their surplus value; they turn out their pockets and cannot find a penny to buy the stored 4666 in means of production" (Luxemburg (1913), p. 93).

Later on, Luxemburg synthesises her point of view on Bauer's model:

"Bauer uses the following tricks to get himself out of this mess. Firstly, he fabricates the "sale" of unsaleable remainder of commodities from Dept. I to Dept. II, without a single word about how the latter pays for it. Secondly, after he fabricated "sale", he lets capitalists II do something even more original: with the newly acquired means of production they walk out of their own department into the other and invest them there as capital; and thirdly, they take with hem their own unsaleable consumer goods, likewise to invest them in the other department as variable capital" (*ibid.* p. 95).

The impossibility to finance the purchase of section II to section I allows Luxemburg to make a judgment, which we consider as correct, on Bauer's model:

"The fact remains that the manipulations of Bauer's capitalists are sheer swindles. These gentlemen pretend to be buying and selling 4666 in means of production, but in reality there are no means with which to buy them. When capitalists I give the remainder of their commodities to capitalists II it is a lovely birthday present. And, in order to act shabbily, capitalists II reply to this noble gesture with equal high-mindedness; they give the present straight back to their colleagues and event generously add their own remainder of consumer goods worth 1167 (they would not know what to do with it, anyway). There you are, folks, take it, God bless you, there you have the variable capital to set your superfluous machines in motion" (*ibid.* p. 94).

Capital transfers are therefore not motivated by the will to accumulate more and more money-capital but by the fact that all factors of production must be fully used, given the rise in the global organic composition of capital. Bauer's accumulation schemes do not take place in a capitalist mode of production.

A. <u>The economic planning of the extended reproduction:</u>

Bauer suggested to integrate capital transfers in order to answer to Luxemburg's problem. To this end, he built a model where accumulation is directed by "*social organs responsible for the planning of production*" (Bauer (1913), p. 92). Similarly he writes:

> "*Let us assume that in a socialist economy….*" (*ibid.*).

In this context, capital transfers are not motivated by the will to accumulate more and more money-capital, via the exchange, but by the obligation to invest all factors of production "*in which the accumulated part of surplus value is embodied*", given the increase in the global organic composition of capital.

A part of the surplus value is therefore not realised but directly transferred. It is because it is impossible, in the schemes of extended reproduction, to represent the exchange of goods against money, that Bauer can write:

> "*not only in the first year but also in every subsequent year the entire value of the product of both departments is sold without any disturbance, and the total surplus value is realised. Comrade Luxemburg's hypothesis that the accumulated part of the surplus value could not be realised is thus false*" (*ibid.* p. 100).

Thereby, when Bauer writes:

> "*Thus it is not at all strange that Rosa Luxemburg cannot realise (κ + α)* [surplus value] *in the first year, for she assumes that capitalists buy the material elements of additional productive capital only in the second year. (...) The whole difficulty arises only if it is assumed that the selling period of commodities in which the accumulated surplus value is embodied is longer than the selling period of other commodities*" (*ibid.*, p. 103),

it seems logical that Luxemburg's answer is quite virulent:

> "*That is the heart of the matter. I was not aware that, if one wanted to open a factory and put it into production in 1916, one had to construct the necessary buildings, buy the machines and materials and get the provisions in stock for the workers who are to be employed – in 1915. I was under the impression that one founds a business first and then buys the building site for it, that one employs workers first and then plants the rye which will be baked into bead for them! (…) Marx was dealing with the economic metamorphoses of products and their connexion in a capitalist economy; he was dealing with the fact that, in a*

capitalist world, the sequence of the economic processes is: production –
exchange – consumption – production again – exchange – consumption, ad so on
in an endless chain" (Luxemburg (1913), p. 101).

Consequently, the impossibility of the extended reproduction of the capitalist mode of
production cannot be excluded. And for now, the impossibility of the extended reproduction
in a planned economy can be excluded. It is an implicit criticism that we can find in
Luxemburg's writings:

> "*As one can see, everything depends on whether the capitalists "want" to*
> *undertake an expansion of production. And why not? Well, of course, "they want*
> *to"!(...) But the question arose as to whether the capitalists, who of course always*
> *"want" to accumulate, can also do so, that is whether they can find continually*
> *expanding market for expanded production, and where such a market is to be*
> *found?*" (*ibid.* p. 67);
>
> "*To accumulate capital does not mean to produce higher and higher mountains of*
> *commodities, but to convert more and more commodities into money capital*"
> (*ibid.* p. 71).

She does not accept that the 4666 (at time $t = 0$) of surplus value bought and transferred to
sector I, represents the realisation of a part of the surplus value in sector II:

> "*Bauer appeals to the fact that, with technological progress, the production of*
> *means of production will grow at the expense of the consumer goods production,*
> *and the capitalists in the latter department will thus constantly place a portion of*
> *their surplus value in the former department in some form or other. (…) All this is*
> *excellent. However, the "transfer" of accumulated surplus value from one branch*
> *of production to another can only occur in the form of money capital, that form of*
> *capital which does not differentiate and is absolute, and is therefore essential for*
> *social fluctuation, to initiate the displacements of social commodity production.*
> *(…) The point was to show how general exchange converts capitalist commodities*
> *into money capital, which alone enables the fluctuation from one branch of*
> *production to the other*" (*ibid.* p. 96).

Rosa Luxemburg is not wrong except in the fact that she thinks that Bauer's model describes
a capitalist economy. On the other hand, Bauer is wrong thinking his model is an answer to
Luxemburg's problem. He only demonstrated the possibility of the extended reproduction in a

socialist environment. However, Luxemburg is not very shrewd when she describes Bauer's model as a "*shapeless mash*" (*ibid*. p. 97), because, first, the economy would be directed by "*social organs responsible for the planning of production*", and, second, because the remainder of consumption goods could not be used for the production of capital goods:

> "*A load of unsaleable wax candles cannot buy shares in copper mines, nor can a warehouse full of unmarketable rubber shoes set up a new machine factory*" (*ibid.* p. 96),

but one can pay workers in consumption goods to work with unsaleable machines to the production of new means of production. This, however, implies to analyse the evolutions of the sectoral organic compositions of capital.

B. <u>Market clearing and the sectoral organic compositions of capital:</u>

Even if Bauer and Luxemburg agree to say that the growth of the global organic composition of capital characterises the development of productive forces, this concept is differently interpreted in the writings of the two authors.

In Luxemburg's works, the continuing rise of the global organic composition of capital is materialised in the sectoral organic compositions of capital. It is a technical data representing the mechanisation of production. In this model[8], where capital transfers are excluded, the rate of surplus-value increases in time. Capitalists' rate of savings out of profits is 0,5. This surplus value in invested in constant and variable capitals in a ratio of 6/1, then 7/1… We can now describe the evolutions of C_j (t) and V_j (t), j = I, II. The rate of surplus value is 100% at time t = 0 and increases by 1/34 at each period[9]. Consequently, the evolutions of constant and variable capitals in each sector can be known:

$$V_I(t) = V_I(t\text{-}1) + (s).\left(\frac{1}{t+5}\right).\left(1+\left(\frac{t}{34}\right)\right).\left[V_I(t\text{-}1)\right] = V_I(t\text{-}1)\frac{(69t+380)}{(68t+340)} \; ;$$

avec $V_I(0) = 1071$.

---

[8] Luxemburg (1913a, p. 333).

[9] This formalisation is only illustrative. It is used to present the development of the productive forces in Luxemburg's work. As we will understand in the next section, it is certainly not possible to make both assumptions, at the same time, of a determinate rate of surplus value and of a determinate rise of the global organic composition of capital. For that matter, Luxemburg, even if she makes the assumptions of a rising rate of surplus value, does not tell us its evolution over time.

$$V_{\text{II}}(t) = V_{\text{II}}(t\text{-}1)\ \frac{(69t + 380)}{(68t + 340)}\ ;$$

avec $V_{\text{II}}(0) = 311$.

$$C_{\text{I}}(t) = C_{\text{I}}(t\text{-}1) + (s).\left(1-\left(\frac{1}{t+5}\right)\right).\left(1+\left(\frac{t}{34}\right)\right).[V_{\text{I}}(t\text{-}1)]$$

$$= C_{\text{I}}(t\text{-}1) + V_{\text{I}}(t\text{-}1)\frac{(t^2 + 34t + 148)}{(68t + 340)}\ ;$$

avec $C_{\text{I}}(0) = 5428$.

$$C_{\text{II}}(t) = C_{\text{II}}(t\text{-}1) + V_{\text{II}}(t\text{-}1)\ \frac{(t^2 + 34t + 148)}{(68t + 340)}\ ;$$

avec $C_{\text{II}}(0) = 1587$.

Graphically, we notice that sectoral organic compositions of capital rise at the same pace but for different values:

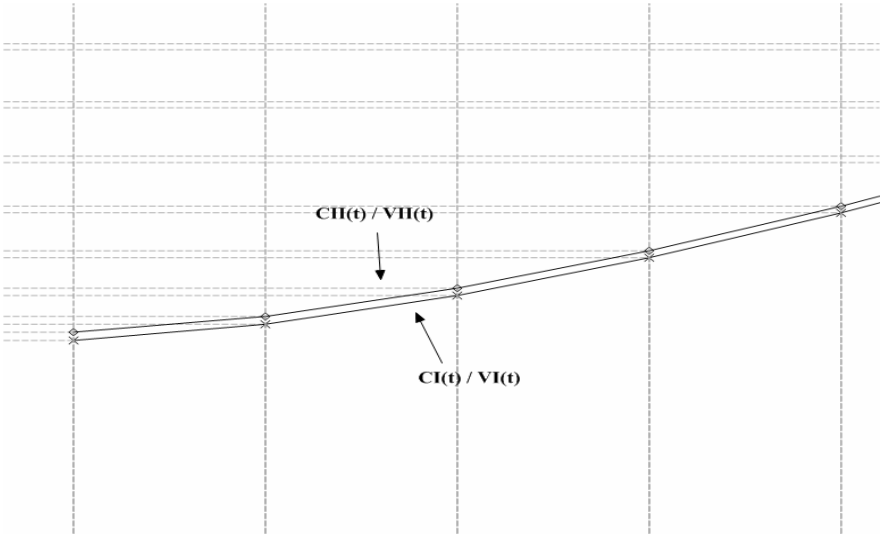Cj(t) / Vj(t) avec taux de plus-value croissant
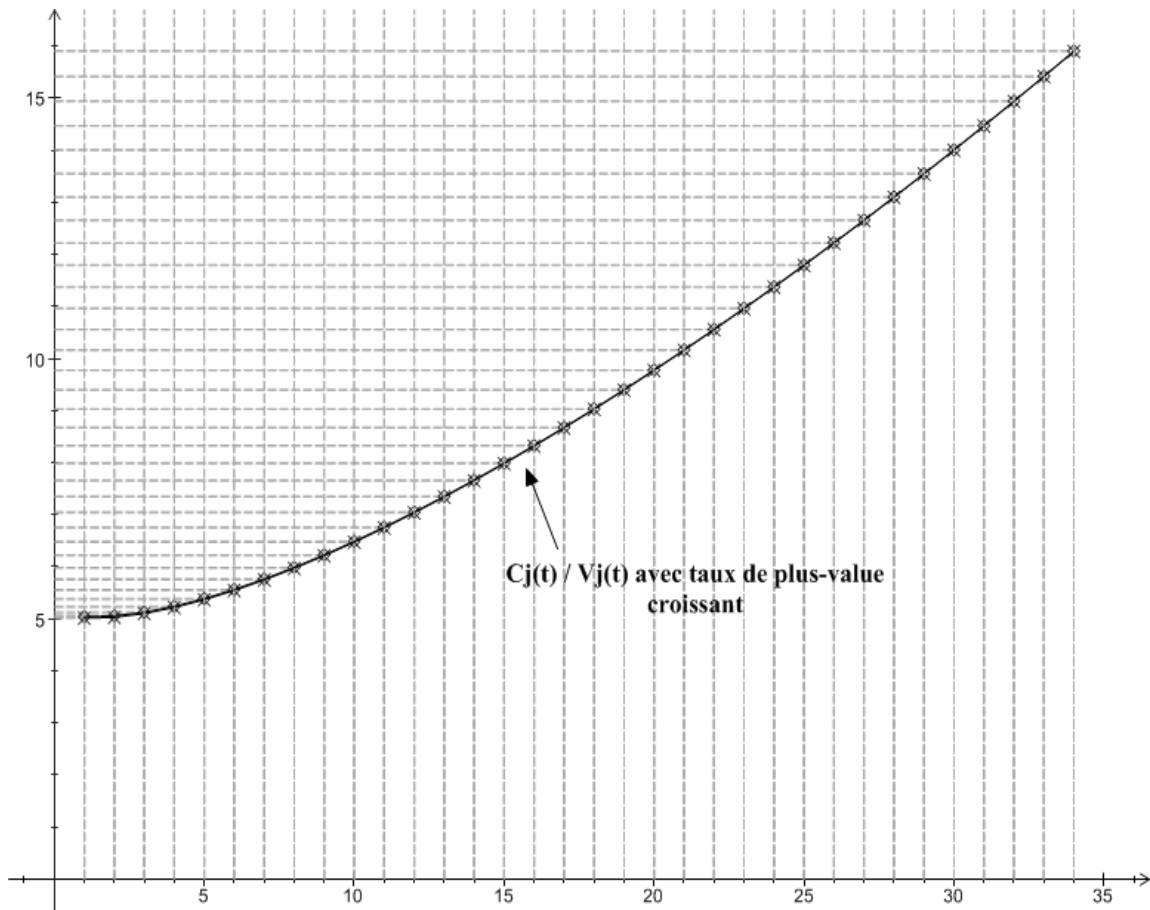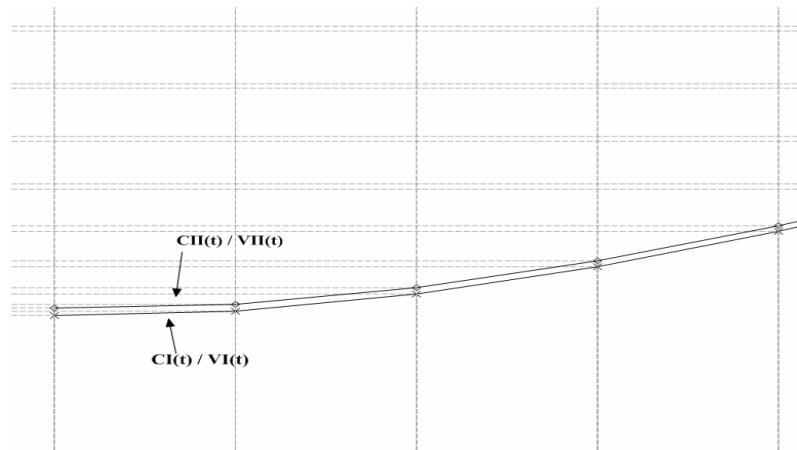
If the rate of surplus value is kept constant, as in Bauer's model, sectoral organic compositions of capital still rise at the same pace, different from the one found when the rate of exploitation is raising. Graphically,
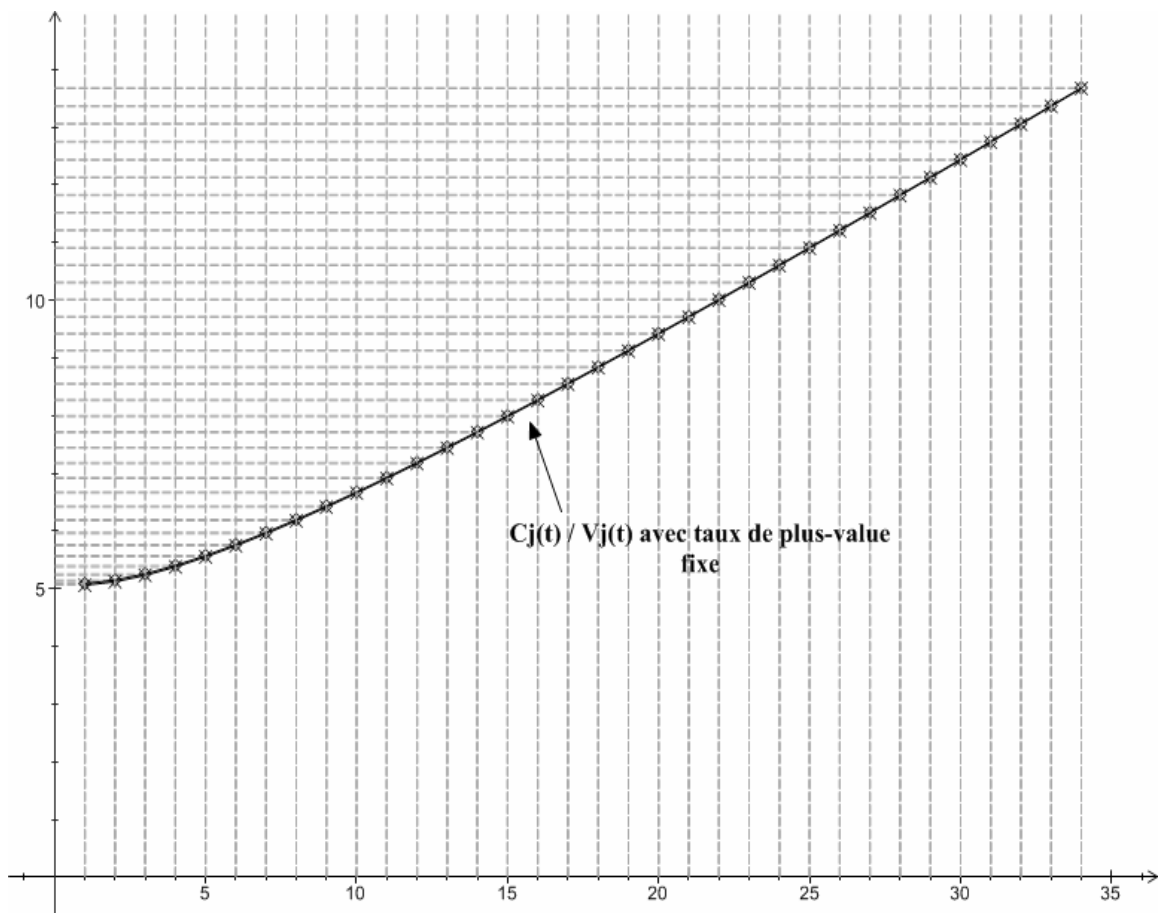
Sectoral organic compositions of capital remain different,

In Bauer's model, sectoral organic compositions of capital adapt to the transfers necessary in order that supplies equal demands on every markets whereas the global organic composition of capital grows. They are endogenous and do not materialise the development of the productive forces. Indeed, even if the ratios between net investments in constant and variable capitals in each department of production are equal to the ratio between constant capital and variable capital newly accumulated in the economy, this does not mean that sectoral organic compositions of capital will rise at the same rhythm as the global organic composition of capital. Graphically,

ICI(t) / IVI(t) = ICII(t) / IVII(t) = alpha C(t) / béta V(t)
Bauer

CI(t) / VI(t)
Bauer

CII(t) / VII(t)
Bauer
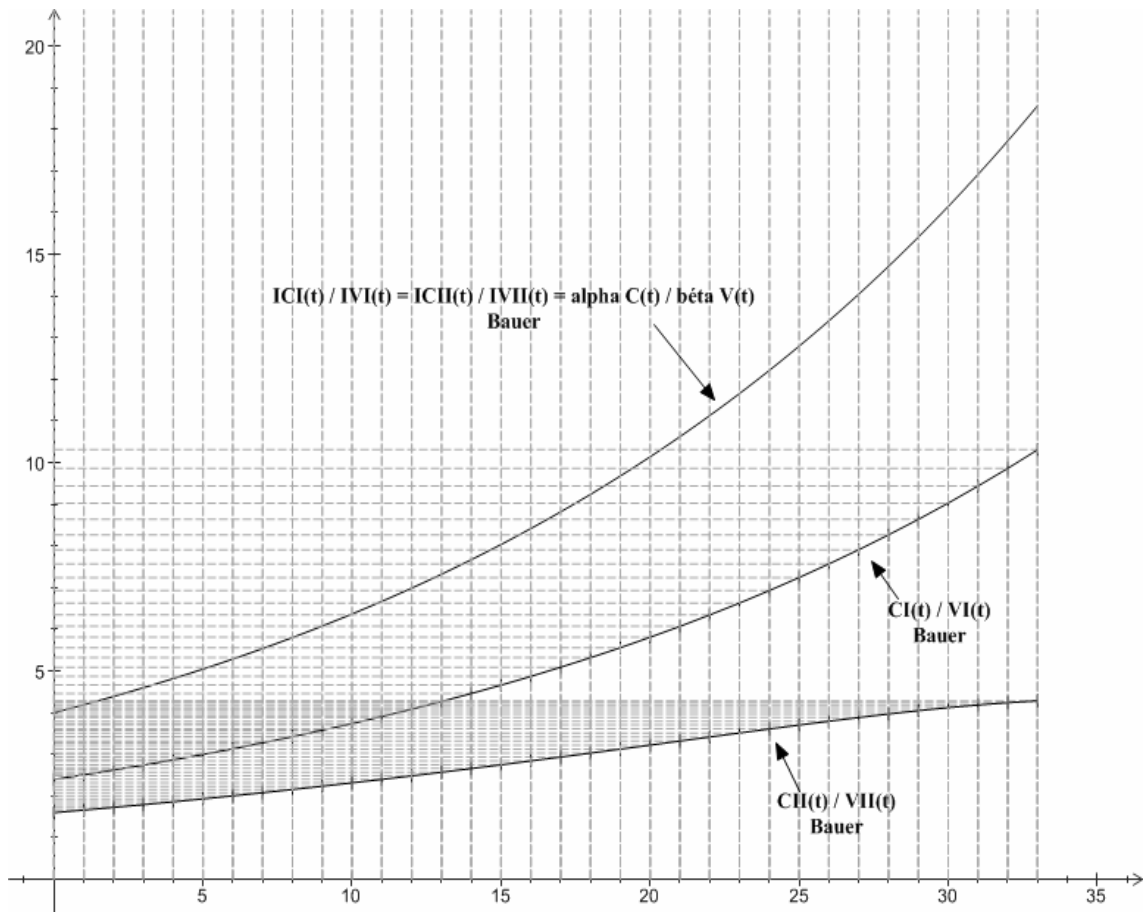
This last point is important because it is on this basis that Luxemburg qualifies Bauer's model as a "*shapeless mash*". It is in fact a different manner to define the development of the productive forces. In Bauer's works, the rhythm of the increase of the global organic composition of capital is different from the ones of the sectoral organic compositions of capital. In Luxemburg's writings, the development of the productive forces is materialised by the increase of the global organic composition of capital and by the rise, at the same rhythm, of the sectoral organic compositions of capital.

C. Crisis theories:

Crisis in a capitalist economy, would be, for Bauer (1913), of two types: of underaccumulation and of overaccumulation. They are linked, it is novelty in Marxist literature, to the growth of the population. The last should grow by 5% per period. In a state of equilibrium the rate of growth of the population is equal to the one of variable capital. We first present these theories before commenting on them. Let's begin by the underaccumulation crisis.

A decline in the rate of savings out of profits of the capitalists is associated to a decline in the accumulation of variable capital[10]:

> "*If the increase in the rate of accumulation falls behind this requirement, the growth of variable capital will lag behind the increase in those seeking work. The condition which then arises can be termed underaccumulation*" (*ibid.*, p. 104).

Consequently, according to Bauer, the "*reserve army*" increases and creates a pressure on wages[11]. This implies a rise in the rate of surplus value which should result in a rise in the accumulation of variable capital[12] until the rate of growth of variable capital equalises the one of the population. If wages are considerably lowered, or if the saving rate increases, the economy will enter, according to Bauer, a state of overaccumulation characterised by a growth rate of variable capital higher than the one of the population[13]. In this case, wages should rise, the rate of surplus value should decline until the rate of growth of variable capital equalises the one of the population. It is therefore not a proportion crisis, defining the imperialism of capitalism, as in Luxemburg, but a moment of the economic cycle:

> "*Viewing the capitalist world as a whole, the tendency for accumulation to adjust to population growth is apparent in the industrial cycle. Prosperity is overaccumulation, which destroys itself in the crisis. The ensuing depression is a time of underaccumulation which also brings itself to an end, inasmuch as the depression itself produces the conditions for renewed prosperity. The periodic alteration of prosperity, crisis, and depression is the empirical expression of the fact that the mechanism of the capitalist mode of production automatically generates overaccumulation and underaccumulation, with the accumulation of capital adjusting again and again to the growth of population. (...) There exists in the capitalist mode of production a tendency for the adjustment of capital accumulation to the growth of population*" (*ibid.*, p. 106-107).

On the other hand, a socialist economy would not enter a crisis:

> "*in a socialist society the social organs responsible for the planning of production ensure that the expansion of productive capacity and food supplies keeps pace with population growth*" (*ibid.*, p. 92).

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[10]This case is arbitrary. A decline in the rate of savings can also be associated to a decline in the accumulation of constant capital.

[11] This theory preempts Goodwin's proposition (1967). When the reserve army grows, wages go down and *vice versa*.

[12] The growth of the rate of exploitation could also result in a rise in the accumulation of constant capital, intensifying the crisis.

[13] This case is also arbitrary. A decline in wages or a rise in the rate of saving can also be associated to a rise in the accumulation of constant capital.

Bauer's crises could only happen in a capitalist economy in which agents would be free to invest capitals whereas they would be excluded, by definition, if the economy is directed by social organs. But if the capitalists are free to invest wherever they want, it would certainly be in the department which is the more profitable. But in Bauer's model, the transfers are made from the sector with the highest rate of profit to the sector with the lowest one. If production is not lead by social organs, the possibility of the extended reproduction can be questioned and Bauer's crises theories must be abandoned.

Luxemburg is therefore wrong to study the possibility of the extended reproduction of a capitalist economy without taking into account competition[14] whereas Bauer is wrong to think that if she had taken into account capital transfers in a capitalist economy, the extended reproduction would become possible. This is only the case, for now, of a socialist economy.

There can, however, exist an other type of crisis in Bauer's model. Bauer did not notice it because his study is developed only over 4 periods. It comes from Bauer's assumptions. As we noticed, at time $t = 34$, there will not be enough saving to make the global organic composition of capital growing such that constant and variable capitals rise, respectively, by 10% and 5%. The imposed rise of the global organic composition of capital will be made possible only at the expense of a decline in the production of consumer goods.

At period $t = 33$ we have,

	Sector I (Capital goods)	Sector II (Cons. goods)	Total
C_j (33)	4279515,982	365514,902	4645030,884
V_j (33)	415008,995	85309,859	500318,854
DK_j (33)	8958,335	1841,489	10799,824
AC_j (33)	385300,21	79202,878	464503,088
AV_j (33)	20750,45	4265,492	25015,943
Total	5109533,972	536134,62	

with,

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[14] "*But there must obviously be invisible rules which somehow work in al this chaos of competition and anarchy, otherwise capitalist society would have been in ruins long ago. (…) But price instability and crises have only one function in society: to integrate chaotic private production into its broad general context, without which it would soon disintegrate. Et us here try to sketch, with Marx, the relation between total capitalist production and social needs. We will only omit the specific capitalist methods of price fluctuations and crises, and concentrate on the basics*" (*ibid*. p. 54).

IC$_j$ (33)	461269,774	3233,314	464503,088
IV$_j$ (33)	24841,811	174,131	25015,943

At time $t = 34$, the capitalists will accumulate and invest all their surplus value but this is not enough to allow the global organic composition of capital to rise in such a way that gross investments in constant and variable capitals increase, respectively, by 10% and 5%. On the other hand, net investments in constant and variable capitals in department II a time $t = 34$ will still be function of $\dfrac{(1 + \mu)\, \beta V(t) - \alpha^2 C(t)}{(1 + \mu)\, \beta V(t) + \alpha C(t)}$ where $V(t) \neq V(0)\,(1 + \beta)^t$ et $C(t) \neq C(0)\,(1 + \alpha)^t$. Net investments in constant and variable capitals can be rewritten as follow,

$$IC_I\,(t) = AC\,(t) - IC_{II}\,(t)\,;$$

$$IV_I\,(t) = AV\,(t) - IV_{II}\,(t).$$

In parallel, sectoral and global accumulations in constant and variable capitals cannot be made in a ratio evolving in accordance with the function $\dfrac{\alpha C(0)\,(1+\alpha)^t}{\beta V(0)\,(1+\beta)^t}$. If it were the case, markets would not be cleared, and there would be systematically an excess of supply of capital goods and an excess of demand of consumer goods.

Therefore, with the rate of saving out of profits equals to 1,

$$AC\,(t) = C_I\,(t) + 2\,V_I\,(t) - C(t)\,;$$

$$AV\,(t) = C_{II}\,(t) + 2\,V_{II}\,(t) - V\,(t).$$

$$AC_j\,(t) = \frac{AC(t)}{V(t)}\,.[V_j\,(t)]\ (j = I, II);$$

$$AV_j\,(t) = (1 - \frac{AC(t)}{V(t)})\,.[V_j\,(t)]\ (j = I, II).$$

At time $t = 34$, we obtain,

	Sector I (Capital goods)	Sector II (Cons. goods.)	Total
C$_j$ (34)	4740785,756	368748,216	5109533,972
V$_j$ (34)	439850,806	85483,99	525334,797
DK$_j$ (34)	0	0	0
AC$_j$ (34)	427809,588	83143,807	510953,396
AV$_j$ (34)	12041,218	2340,183	14381,401
Total	5620487,368	539716,196	

with,

IC$_j$ (34)	509646,611	1306,784	510953,396
IV$_j$ (34)	14314,223	67,178	14381,401

The capitalists accumulate and invest the maximum of constant and variable capitals. In this context, net investments in constant and variable capitals in department II are positive. They represent, however, necessary net investments in sector II in order to satisfy the variation in the demand for consumer goods. The last is still positive but declines over time.

At time $t = 35$, we have,

	Sector I (Capital goods)	Sector II (Cons. goods)	Total
C$_j$ (35)	5250432,367	370055	5620487,367
V$_j$ (35)	454156,029	85551,168	539716,196
DK$_j$ (35)	0	0	0
AC$_j$ (35)	452952,328	85322,731	538275,058
AV$_j$ (35)	1212,701	228,437	1441,14
Total	6158762,425	541157,336	

with,

IC$_j$ (35)	540312,649	-2037,59	538275,058
IV$_j$ (35)	1539,018	-97,88	1441,14

The fact that net investments in constant ad variable capitals in department II are negative signifies that the growth of the global organic composition of capital will be obtained, not by a rise of gross investments in constant and variable capitals of, respectively, 10% and 5%, but by a rise of gross investments in constant capital and a decline of gross investments in variable capital. However, the fact that net investments in constant and variable capital in sector II were positive at the preceding stage implies that the production of consumer goods has risen from period $t = 34$ to period $t = 35$. Even if department II disinvests, there is still more variable capital than before. It is used to increase investments in variable capital in department I. The disinvestment, in proportion, in constant capital in sector II, being higher than the disinvestment in variable capital in this sector, implies that the organic composition of capital of this sector will decline. The one department I still continues to increase.

At time $t = 36$,

	Sector I (Capital goods)	Sector II (Cons. goods)	Total
C_j (36)	5790745,016	368017,41	6158762,426
V_j (36)	455703,997	85453,34	541157,337
Total	6702153,01	538924,09	

with,

IC_j (36)	550258,964	-6868,38	543390,314
IV_j (36)	-1931,492	-301,755	-2233,247

From time $t = 35$ to time $t = 36$, the production of consumer goods has declined and the one of capital goods has increased. This is due to the fact that net investments in constant and variable capital at the preceding period were negatives. At time $t = 36$, because of the decline in the production of consumption goods, we cannot speak anymore of a transfer of variable capital from sector II to sector I. There is a disinvestment transfer in variable capital.

Calculating AV_{II} (36) $= 85543,34 \cdot \left(\dfrac{-2233,247}{541157,337} \right) = -352,694$, we understand that part of the

disinvestment in variable capital in department II, 352,694 - 301,755 = 50,939, will be supported by department I.

Proportionally, the disinvestment in constant capital in sector II is more important than its disinvestment in variable capital, its organic composition of capital continues to decline. The one of sector I continues its growth.

To conclude, net investments in constant and variable capitals in department I, and consequently in department II also, are represented by functions (1) and (2), which are functions, among other, of the variation in the demand for consumer goods at the next period, $(1 + \mu) \beta V(t) - \alpha^2 C(t)$. However, at time $t = 34$, the amount of surplus value is not sufficient to allow the growth of the global organic composition of capital in such a way that gross investments in constant and variable capitals rise, respectively, by 10% and 5%. The decline in capitalists' consumption is substituted, at first, by the decline of the growth rate of the production of consumer goods, and, then, by a decline of the production of consumer goods. It is therefore department II which finances the rise in the global organic composition of capital. The production of capital goods will increase, but more and more slowly, and the production

of consumption goods will decline more and more quickly. From time $t = 36$, the rate of growth of the global organic composition of capital will never cease to increase, until the disappearance of department II, presumably because of a lack of constant capital in this sector[15].

III. Boundless accumulation and the rate of exploitation:

In Bauer's model, the capitalists have the same rate of savings out of profits, the rate necessary to satisfy the desired growth of the global organic composition of capital. Sectoral demands being fixed by assumptions, there must be capital transfers, the organic composition of capital increasing in such a way that gross investments in constant and variable capitals rise, respectively, by 10% and 5%, from the department producing consumer goods to the department producing capital goods. In other terms, the supply of each sector adjusts to its demand and markets are cleared at every period.

We have showed:
1) that all the surplus value, in Bauer's model, cannot be realised. Consequently, the critique of Luxemburg to Bauer is justified;
2) that Bauer locates his model in a socialist environment and not in a capitalist one. The part of surplus value which is not realised is therefore directly transferred, without compensation;
3) that even if the ratios between net investments in constant and variable capitals in sector I and II are equal to the ratio between constant capital and variable capital newly accumulated in the economy, sectoral organic compositions of capital vary differently. They do not represent the mechanisation of production. They evolve and bring the clearing of markets in consequence of capital transfers necessary to satisfy the desired growth of the global organic composition of capital;

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[15] Orzech & Groll (1983) suggest to know the conditions under which the economy can be in a state of balanced growth. They assume that $\beta = \alpha$ and find that the rate of savings out of profits is constant and that organic compositions of capital will converge to the value of the social organic composition of capital. In this sense, Bauer's model anticipates Harrod's model. A rise in the global organic composition of capital implies a rise in the rate of savings out of profits.

4) that the crisis theories in a capitalist economy suggested by Bauer should be abandoned, because it integrates capital transfers between sector which are understandable only in a socialist economy;

5) that there can exist another crisis in the extended reproduction schemes, even in a socialist economy. It comes from the fact that, after a finite number of periods, the rate of exploitation is not high enough to satisfy the desired growth of the global organic composition of capital.

The last point was commented by Luxemburg:

" *Now Otto Bauer, unlike Marx, takes good note of technological progress in his tables, and incorporates it into his calculation in the most explicit way, so that he lets constant capital grow twice as fast as variable capital from year to year. Indeed, as he expounds his theory further he assigns a determining role to technical progress in the variation in the state of business. But what de we see over the page? In the same breath Bauer assumes a fixed and constant rate of surplus value "to simplify the investigation". (…) Of course, in all his models of reproduction Marx assumes a permanently fixed rate of surplus value, and one can hold that this very assumption is not legitimate for the investigation of the problem of accumulation. Marx, however, did stick rigorously to his assumption, and within the limits of that assumption: he ignored technological progress in every case. Bauer treats the subject quite differently: like Marx he assumes a fixed rate of surplus value; but unlike Marx he simultaneously assumes strong and continuous technological progress! He brings technological progress into his calculation, but this by no means raises the level of exploitation – two conditions which completely contradict and neutralize one another*" (Luxemburg (1913), p. 98-99);

"*But these conditions themselves are quite "astonishing". For as long as we are not wandering around in this air but standing on the surface of this capitalist earth, what incentive do the capitalists have to make use of technological progress and to invest even larger sums in constant capital is the whole benefit is only for the working class? According to Marx, the creation of "relative surplus value", the increase in the rate of exploitation through the cheapening of the labour force, is the only objective reason for the capitalist class as a whole to promote technological progress; it is the real objective result of the competition of*

individual capitals, aiming unconsciously for extra profit. Bauer's astonishing assumption is a pure economic impossibility as long as capitalism exists" (*ibid*. p. 109).

We have to remind here that Bauer treats exclusively of a planned economy. In this context, a fix rate of surplus value is understandable. It still remains the cause of the macroeconomic crisis.

A. Bauer's model with a raising rate of surplus value:

This project is called upon by Luxemburg:

"*It is a pity that Bauer did not consider it worth his trouble to go on to complete this little detail himself, instead of breaking off his ingenious calculations, just lie the other calculation experts, and taking leave of us because of urgent delays at the very point where his proof should have begun. This at least would be the only way in which a arithmetical "proof" could have been provided for Bauer's assertion. What he has now provided is no longer an aid for scientific analysis, but quackery, which explains nothing and can prove nothing*" (Luxemburg (1913), p. 99).

It will be however showed that the growth of the rate of surplus value in Bauer's model makes it incoherent. Bauer does not take a fixed exploitation rate to "*simplify*" his study but because it would be impossible to do otherwise.

The macroeconomic crisis occurs at time $t = 34$ because of a lack of saving if the global organic composition of capital must rise in such a way that gross investments in constant and variable capitals increase, respectively, by 10% and 5%, capitalists consumption being residual. This limit could be lifted up by allowing the rate of surplus value to increase continuously. Let's call $\mu_j(t)$ the rate of surplus value in sector j (j = I, II) at time t and $\mu_j(t+1)$ the rate of surplus value in the same sector at time $t + 1$. Let's assume that $\mu_j(t+1) > \mu_j(t)$.

The variation of total demand for capital goods does not depend on the rate of surplus value and is always positive:

$$P_I(t+1) - P_I(t) = C(t)(\alpha + \alpha^2).$$

Net investments in constant and variable capitals in department I will always be positive and their level will depend on the rate of surplus value:

$$IV_I(t) = \beta V(t) \frac{C(t)\,(\alpha + \alpha^2)}{[1 + \mu_I(t+1)]\,\beta V(t) + \alpha C(t)} \qquad (5),$$

$$IC_I(t) = \alpha C(t) \frac{C(t)\,(\alpha + \alpha^2)}{[1 + \mu_I(t+1)]\,\beta V(t) + \alpha C(t)} \qquad (6),$$

with, by assumptions,

$$V(t) = V(0).(1 + \beta)^t \text{ and,}$$
$$C(t) = C(0).(1 + \alpha)^t.$$

Let's now rewrite the variation in the total demand of consumer goods in function of the rate of surplus value. The total demand in consumer goods at time t is:

$$P_{II}(t) = V(t) + \beta V(t) + DK(t)$$

and,

$$DK(t) = \Pi(t) - \beta V(t) - \alpha C(t) = \mu(t)V(t) - \alpha C(t) - \beta V(t).$$

This implies that,

$$P_{II}(t) = V(t) + \mu_I(t)V_I(t) + \mu_{II}(t)V_{II}(t) - \alpha C(t).$$

At the next period,

$$P_{II}(t+1) = (1 + \beta)V(t) + \mu_I(t+1)V_I(t+1) + \mu_{II}(t+1)V_{II}(t+1) - \alpha(1 + \alpha)C(t).$$

Recalling that,

$$V_j(t+1) = V_j(t) + IV_j(t)\ (j=I, II),$$

we obtain,

$$P_{II}(t+1) - P_{II}(t) = \Delta P_{II}(t+1)$$

$$= \beta V(t) + V_I(t)\,[\mu_I(t+1) - \mu_I(t)] + V_{II}(t)\,[\mu_{II}(t+1) - \mu_{II}(t)] + \mu_I(t+1)\,IV_I(t) + \mu_{II}(t+1)\,IV_{II}(t)$$
$$- \alpha^2 C(t).$$

If the markets have to be cleared at every period, the variation of the total supply of consumption goods should equal the variation of the total demand of consumption goods:

$$\beta V(t) + V_I(t)\,[\mu_I(t+1) - \mu_I(t)] + V_{II}(t)\,[\mu_{II}(t+1) - \mu_{II}(t)] + \mu_I(t+1)\,IV_I(t) + \mu_{II}(t+1)\,IV_{II}(t)$$
$$- \alpha^2 C(t)$$
$$= IC_{II}(t) + IV_{II}(t)\,[1 + \mu_{II}(t+1)],$$

or,

$$\beta V(t) + V_I(t)\,[\mu_I(t+1) - \mu_I(t)] + V_{II}(t)\,[\mu_{II}(t+1) - \mu_{II}(t)] + \mu_I(t+1)\,IV_I(t) - \alpha^2 C(t)$$
$$= IC_{II}(t) + IV_{II}(t).$$

If by assumption,

$$\frac{IC_{II}(t)}{IV_{II}(t)} = \frac{\alpha C(t)}{\beta V(t)},$$

we find,

$$\beta V(t) + V_I(t)\,[\mu_I(t+1) - \mu_I(t)] + V_{II}(t)\,[\mu_{II}(t+1) - \mu_{II}(t)] + \mu_I(t+1)\,IV_I(t) - \alpha^2 C(t)$$

$$= IV_{II}(t)\left(\frac{\alpha C(t) + \beta V(t)}{\beta V(t)}\right).$$

If, also by assumption,

$$\beta V(t) = IV_{II}(t) + IV_I(t),$$

we obtain,

$$\beta V(t) + V_I(t)\,[\mu_I(t+1) - \mu_I(t)] + V_{II}(t)\,[\mu_{II}(t+1) - \mu_{II}(t)] + \mu_I(t+1)\,IV_I(t) - \alpha^2 C(t)$$

$$= [\beta V(t) - IV_I(t)]\left(\frac{\alpha C(t) + \beta V(t)}{\beta V(t)}\right).$$

This implies,

$$IV_I(t) = -\left[\beta V(t)\frac{V_{II}(t)[\mu_{II}(t+1) - \mu_{II}(t)] + V_I(t)[\mu_I(t+1) - \mu_I(t)] - C(t)\,(\alpha + \alpha^2)}{\alpha C(t) + \beta V(t) + \beta V(t)\mu_I(t+1)}\right].$$

But this definition of the net investment in variable capital in sector II cannot be identical to equation (5) except if,

$$V_I(t)\,[\mu_I(t+1) - \mu_I(t)] + V_{II}(t)\,[\mu_{II}(t+1) - \mu_{II}(t)] = 0.$$

This means that total investment in variable capital cannot be equal to $\beta V(t)$ if the rate of surplus value rises from period to period. If the rate of exploitation were increasing continuously, net investments in variable capital in department I would be excessive. Total net investment in variable capital should be equal to $\beta V(t)$ whereas total net investment in constant capital should be equal to $\alpha C(t)$. In opposition to the views of Brofenbrenner & Wolfson (1984) and Samuelson & Wolfson (1986), a raising rate of surplus value, therefore, contradicts the assumption that markets clear at every period. Moreover, it is not to simplify[16] its study that Bauer assumes a fix rate of surplus value but because it would be impossible to characterise the development of the productive forces as he would have liked to with a continuously rising rate of surplus value. The way Bauer thinks the development of the productive forces is therefore erroneous.

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[16] "*To simplify the investigation we assume for the time being that the rate of surplus value remains unchanged*" (Bauer (1913), p. 93).

The rate of surplus value can only be fix in this model. Consequently the macroeconomic crisis cannot be avoided. Bauer's model, therefore, does not prove the possibility of the extended reproduction[17].

B. <u>An overdetermined model:</u>

We have now to analyse if the rate of surplus value can be fixed at whatever level. The idea behind this change is to know if it is possible to choose a very high rate of exploitation to make the macroeconomic crisis coming at the latest times. However, Bauer makes the assumptions that the capitalists' consumption is residual[18] and that markets clear at every period[19]. It is therefore possible to write:

$$P_{II}(t) - V(t) - \beta V(t) - DK(t) = 0.$$

With,

$$DK(t) = \mu(t)V(t) - \beta V(t) - \alpha C(t), \text{ et,}$$

$$P_{II}(t) = C_{II}(t) + V_{II}(t) [1 + \mu(t)],$$

$$C_{II}(t) + V_{II}(t) [1 + \mu(t)] - V(t) - \mu(t)V(t) + \alpha C(t) = 0,$$

We obtain,

$$\mu(t) = \frac{C_{II}(t) + \alpha C(t)}{V(t) - V_{II}(t)} - 1.$$

It is consequently not possible, in Bauer's model, to have both given α et $\mu(t)$. The assumption of the clearing of the markets at every period imposes to make endogenous the variation of the global organic composition of capital. In Bauer's numerical example, given his assumptions on capital transfers in order that markets clear at every period, $\dfrac{C_{II}(t) + \alpha C(t)}{V(t) - V_{II}(t)}$ is a constant. If the rate of surplus value is equal to 1, Bauer had to find an example where,

$$\frac{C_{II}(0) + \alpha C(0)}{V(0) - V_{II}(0)} = 2.$$

This means that in Bauer's model, C(0), $C_{II}(0)$, V(0) and $V_{II}(0)$ are not arbitrary numbers but are assumptions in the strong sense of the term. If we were questioning why Bauer chose to set C(0) = 200000, he should answer that if it were not the case, given that C_{II} = 80000, that

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[17] Even for a socialist economy.

[18] "*To increase constant capital (from 200000 to 220000) 20000 is required ; to increase variable capital (from 100000 to 105000) 5000 is needed. Thus in total 25000 is accumulated and 75000 is consumed*" (Bauer (1913), p. 93).

[19] "*In the second year the value of the products of the consumer-goods industries must thus amount to 188000, for the consumers goods can only be exchanged against these sums of value*" (Bauer (1913), p. 95).

V(0) = 100000 and that VII(0) = 50000, the rate of surplus value would not be equal to 1. This ambiguity can find an echo in the rhetoric used by Bauer when he presents his assumptions: "*Only the assumptions made in the first year are arbitrary*". The numbers should be arbitrary chosen and the assumptions justified. Here, numbers are not arbitrary, they are assumptions. Bauer cannot, therefore, make us believe, at the same time, that the rate of surplus value is equal to 1 and that the numbers are chosen arbitrarily:

"*Only the assumptions made in the first year are arbitrary : that constant capital amounts to 200000 and variable capital to 180000 ; that variable capital is divided equally between the two departments of production ; that the rate of surplus value is 100 per cent ; and that the rate of accumulation is the same in both departments. Also arbitrary is the assumption that constant capital grows at an annual rate of 10 per cent, and variable capital at 5 per cent*" (Bauer (1913), p. 96);

"*We will assume that constant capital grows at 10 per cent par annum. Thus we obtain, for example, the series*

	C_t	V_t
Period 0	200000	100000
Period 1	$220000 = (1 + \alpha)C_0$	$105000 = (1 + \beta)V_0$
Period 2	$242000 = (1 + \alpha)^2 C_0$	$110250 = (1 + \beta)^2 V_0$
Period 3	$266200 = (1 + \alpha)^3 C_0$	$115762 = (1 + \beta)^3 V_0$

To simplify the investigation we assume for the time being that the rate of surplus value remains unchanged, at 100 per cent... To increase constant capital (from 200000 to 220000) 20000 is required; to increase variable capital (from 100000 to 105000) 5000 is needed. Thus in total 25000 is accumulated and 75000 is consumed" (Bauer (1913), p. 93).

In other words, it impossible that Bauer's model, with Bauer's numbers, works with another rate of surplus value different from 1. If the rate of surplus value is fixed by assumption, the evolution of the global organic composition of capital cannot be fixed by assumption. We know understand better why a continuously rising rate of surplus value cannot be integrated into the model. It would suppose to change all the numbers at every period. In economical terms, this is vain.

Concluding remarks:

At best, therefore, Bauer's model is a particular case. Bauer suggested to show that with a rate of surplus value equal to 1, the extended reproduction of a socialist economy, given a development of the productive forces such that gross investments in constant and variable capitals increase, respectively, by 10% and 5%, was possible thanks to the transfer process. This exercise being not possible with numbers chosen arbitrarily, Bauer selected the ones answering to his problem. They are not arbitraries anymore but are assumptions in order to make function the model.

Finally, let us notice that the same assumptions are present in Luxemburg's accumulation schemes and could be the cause of the impossibility to realise part of the surplus value, and consequently, the cause of the imperialist character of the capitalist mode of production. This will the next stage of our researches.

Bibliography

- Bauer, O. (1913), "The accumulation of capital", *History of Political Economy*, (1986), Vol. 18, n°:1.

- Bronfenbrenner, M. & Wolfson, M (1984), "Marxian macrodynamics and the Harrod growth model", *History of Political Economy*, Vol. 16, n°2.

- Luxemburg, R. (1913), *The Accumulation of Capital – An Anti-critique*, Monthly Review Press, (1972).

- Luxemburg, R. (1913a), *The Accumulation of Capital*, Routledge and Kegan Paul Ltd., London, (1951).

- Orzech, Z. B. & Groll, S. (1983), "Otto Bauer's scheme of expanded reproduction : an early harrodian growth model", *History of Political Economy*, Vol. 15, n°4.

- Samuelson, L. & Wolfson, M. (1986), "Expository Marxism and comparative economic dynamics", *History of Political Economy*, Vol. 18, n° 1.